

1. *Assignment*
 - a. Continue formulating questions about the paper we're outlining in class.
 - b. Continue formulating questions about the social organization of mathematics.
2. *In-class outline.*
 - a. We added some major topics, theorems about bases and dimension.
 - i. I'm concerned that there are a couple of principles still missing that underlie some of these, and whose insertion would affect the order of the outline. We'll discuss those during the next meeting.
 - ii. The question of including infinite-dimensional spaces keeps recurring. I feel that if the proofs are general, they'll be simpler. But that may require pushing more material into the basics part of the introduction.
 - iii. The proof that a basis exists is very short: one line to show that the union of a directed family of independent sets is independent, and a second line to show that Zorn's lemma thus yields a maximal independent subset.
 - iv. We decided to include examples where appropriate.
 - v. The space of real polynomials in x has an obvious countably infinite basis, the powers of x . But for the space of formal power series in x , this is not a basis because a formal power series is not necessarily the sum of a *finite* number of scalar multiples of powers of x .
 - b. We added some detail to the *basics* part of the introduction. We'll *consider* these, but won't necessarily *include* them. This list will have to be augmented each time we add a new major topic.
3. Here's the current state of the in-class outline
 - a. *Introduction*
 - i. ??Should we explain the basics?
 - (1) vector space
 - (2) independent subset
 - (3) spanning subset
 - b. *Theorem.* A subset of a vector space is minimal spanning if and only if it's maximal independent.
 - c. *Definition.* A subset with these properties is called a basis.
 - d. *Theorem.* Any two bases are equinumerous.
 - i. ??Should we restrict here to *finite* bases?
 - e. *Theorem.* Every vector space has a basis.
 - i. ??If we restricted consideration to vector spaces with finite bases at the start, this theorem would not appear here.
 - ii. ??If not, the theorem has a very short proof using Zorn's lemma.
 - f. *Definition.* The dimension of a vector space is the common cardinality of its bases.
 - g. *Examples*

- i. Finite dimensional
 - ii. Infinite dimensional
 - h. *Rank & nullity theorem.* If f is a linear map from one space V to another, then $\dim V = \dim \text{Range } f + \dim \text{Kernel of } f$.
 - i. ??Is that true for infinite dimensional spaces? If not, can we give a counterexample?
 - i. *Theorem.* If A, B are subspaces of V , then

$$\dim A + \dim B = \dim (A \vee B) + \dim (A \cap B).$$
 - j. ??If infinite-dimensional spaces have been excluded, perhaps the extension of the theory to infinite dimensionality should be considered here.
 - k. ??What features of the definition of “vector space” are really required for this theory?
 - i. We can accommodate fields, perhaps noncommutative, of any characteristic. We don’t want to do modules over rings.
 - l. *References*

Baer, Reinhold. 1952. *Linear Algebra and Projective Geometry*. Pure and applied mathematics, 2. New York: Academic Press. [Cited in ??.]
4. *Typesetting.* While I typed the equation

$$\dim A + \dim B = \dim(A \vee B) + \dim(A \cap B).$$

students noticed several unfamiliar conventions and data-entry techniques.

- a. Do you set such a formula inline or display it, as I did? Generally, longish formulas should be displayed, because otherwise, when their context changes, you’ll probably have to decide where to break them between lines. If necessary, this one could best be broken after the $=$ sign; next best, after the second $+$. But it’s better displayed, with some additional space (0.1 inch here) above and below.
- b. $A \vee B$ is the smallest subspace containing both A and B : their *join*.
- c. I make $\vee$ and $\cap$ with *WordPerfect* macros that I wrote years ago: they are merely symbols in another font. Since I remember their names easily—*join* and *cap*—I can enter them very quickly.
- d. I inserted 0.01-inch spaces between $($ and A because otherwise they collide: a defect in *WordPerfect*.
- e. Virtually *always* in mathematical writing, variables are italicized. But enclosing parentheses are virtually *never* italicized.
- f. I do formulas with font changes and minor horizontal and vertical spacing in *WordPerfect*. For stacked formulas such as

$$\sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

I use *MathType*. It’s very easy to use, but not at all flawless.

- g. *LATEX* is required for mathematical typesetting by some publishers, such as the American Mathematical Society. Its output is without doubt about 5%

nicer. But it's much harder to use, and doesn't allow my kind of WYSIWYG editing. I feel that 5% is an obvious tradeoff. Moreover, software developers are now working on web-oriented languages for mathematical typesetting, and I predict that in a few years *LATEX* will be widely regarded as obsolete. Nevertheless, many aggressive colleagues and students want to use it, perhaps because they feel WYSIWYG's for wimps.