

1. *Assignment*
 - a. Continue formulating questions about the social organization of mathematics.
 - b. Continue thinking about a topic for an in-class outline of an expository paper.
2. *Social Organization of Mathematics*
 - a. I passed around an issue of *Mini-Focus*, the MAA local chapter's newsletter, which features the program of its 1 March meeting.
3. *Discussion of Tarski 1955*
 - a. What area of mathematics? This fits in the narrower subject, *lattice theory*.
 - b. The broader subject is *universal algebra*. A group $\langle G, \circ, \iota \rangle$ and a ring with identity $\langle R, +, \cdot, 0, 1 \rangle$ are algebras in this sense: each is a set considered together with one or more binary operations and one or more constants. Universal algebra considers their properties in relationship to logical properties of their axioms. Defined this way, one of the group axioms must be existential: $(\forall x, y) \exists z [x \circ z = y]$, and some of the properties of groups reflect that. But a group can also be defined as $\langle G, \circ, {}^{-1}, \iota \rangle$, with a singular operation ${}^{-1}$ and that axiom replaced by a universal one: $\forall x [x \circ x^{-1} = \iota]$. To what extent does that affect the properties of groups? Complex vector spaces might seem unamenable to this sort of definition, because of the necessity of speaking about complex scalars as well as vectors. But no, you can define them as structures of the type $\langle V, +, \{\cdot_t : t \in \mathbb{C}\}, 0 \rangle$, where each $\cdot_t$ is the singular operation, scalar multiplication by t . For this approach it makes no difference *how many* operations or constants are involved in the definition. This subject was started in the 1930s by Garrett Birkhoff. By the 1940s, it had become common to consider algebraic structures with relations as well as operations: for example, the structure $\langle \mathbb{R}, +, \circ, 0, 1, \leq \rangle$ for the real number system. The term "universal algebra" seemed somewhat limiting, and was broadened to *model theory*, which treats such structures as models of various logical systems. Besides the connections with logic, this subject considers substructures (e.g., subgroups, subspaces), new structures built from old (e.g., direct products), homomorphisms (including linear mappings of vector spaces), isomorphisms, etc. Alfred Tarski can be called the founder of model theory; he was also one of the original editors of the journal *Universal Algebra*.
 - c. Someone asked whether universal algebra includes category theory. Not really, because a categorical approach to group theory, for instance, would eschew speaking of the elements of a group. It would consider mainly the morphisms, and the properties of groups that can be formulated solely in terms of morphisms.
 - d. The discussion of the paper paralleled to some extent that of Smith 2002. One interesting feature is that Tarski's proofs are self-contained. He referred to other literature only for some basic definitions, and for applications of his theorems.