

MATH 800 FOUNDATIONS OF MATHEMATICS
OUTLINE 31

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1. *Homework.* Here is the list of assignments, with weights for the individual items.

<i>Assignments</i>	<i>Points</i>	<i>Comment</i>
BSTTrivial	2	
BSTR11	1	
BSTR15	1	
BSTR19	1	
BSTS2	2	
BSTS3	2	
Stoll1.5.6	2	
Asst2d	2	
E&PR3	1	
E&PS1	2	
E&PS5	2	
Tarski1.6	1	
POSR3	1	
POSS1	2	
POSS4	2	
CLR3	1	
CLR5	1	
CLS2	2	
CLS3	2	
Tarski2.5b	1	
Tarski3.9	1	
Tarski4.16	1	
Tarski5.3	1	
Tarski5.8	1	
ACR3	1	2 of these
ACR4	1	
ACR5	1	
ACS3	2	2 of these
ACS4	2	
ACS6	2	
MPR3	1	2 of these
MPR4	1	
MPR5	1	

<i>Assignments</i>	<i>Points</i>	<i>Comment</i>
MPS2	2	2 of these
MPS3	2	
MPS4	2	
MPS6	2	
MPS7	2	
MPS8	2	
C2R1	1	2 of these
C2R5	1	
C2R6	1	
C2S1	2	1 of these
C2S13	2	
C2S3	2	1 of these
C2S4	2	
C2S5	2	
C2S6	2	
C2S7	2	1 of these
C2S8	2	
Tarski2.12-13	1	
Tarski2.15b	1	
BL1	1	
BL2	1	
Fell Swoop	1	
BL4	1	
BL5	1	
BL6	1	
TotalPoints	63	

2. In class, the “Boolean Logic” unit

- a. Discuss truth tables
 - i. satisfiable
 - (1) tautologous
 - (2) contingent
 - ii. unsatisfiable = contradictory
 - iii. definitions in terms of $\&$, $\vee$, $\neg$ of $\Rightarrow$, $\Leftrightarrow$, $\neq$, $\uparrow$, $\downarrow$
 - iv. the last two exemplify use of De Morgan laws
 - v. other familiar laws, too, are tautologies : no need to say much
- b. Stoll [1963] 1979 exercises.
 - i. 4.5.12. Determine the validity of the following argument:

If Smith wins the nomination, he will be happy, and if he is happy, he is not a good campaigner. But if he loses the nomination, he will lose the confidence of the party. He is not a good campaigner if he loses the confidence of the party. If he is not a good campaigner, then he should resign from the party. Either Smith wins the nomination or he loses it. Therefore, he should resign from the party.

An argument is *invalid* if its hypotheses and the negation of its conclusion are simultaneously satisfiable.

Set N = “Smith wins the nomination”
 H = “He will be happy”
 C = “He is a good campaigner”
 P = “He will lose the confidence of the party”
 R = “He should resign”

The premisses and conclusion are

- | | |
|------------------------|-----|
| $N \Rightarrow H$ | (1) |
| $H \Rightarrow \neg C$ | (2) |
| $\neg N \Rightarrow P$ | (3) |
| $P \Rightarrow \neg C$ | (4) |
| $\neg C \Rightarrow R$ | (5) |
| $N \vee \neg N$ | (6) |
| R | (7) |

Can we satisfy (1–6, $\neg$ 7) simultaneously? Try:

- | | |
|---|------|
| By ($\neg$ 7, 5), $\neg C$ would be false. | (8) |
| By (8, 4), P would be false. | (9) |
| By (9, 3), $\neg N$ would be false. | (10) |
| By (10), N would be true. | (11) |
| By (11, 1), H would be true. | (12) |
| By (12, 2), $\neg C$ would be true. | (13) |

(8, 13) are contradictory, hence we cannot satisfy the hypotheses and the negation of the conclusion simultaneously, hence the argument is valid.

- ii. 4.5.13.a. I wanted to do this in class, but time didn’t permit. You should work it out. Are these sentences simultaneously satisfiable?

$$A \Rightarrow \neg [B \& C]$$

$$\begin{aligned}
 &[D \vee E] \Rightarrow G \\
 &G \Rightarrow \neg[H \vee I] \\
 &\neg C \ \& \ E \ \& \ H
 \end{aligned}$$

iii. 4.5.13.b. Are these sentences simultaneously satisfiable?

$$\begin{aligned}
 &[A \vee B] \Rightarrow [C \ \& \ D] & (1) \\
 &[D \vee E] \Rightarrow G & (2) \\
 &A \vee \neg G & (3)
 \end{aligned}$$

This isn't so easy to start, because there's no implication to negate. There are three possibilities for making an implication true, but only two for the disjunction. Therefore, I'll start with (3): suppose A is true. Then $A \vee B$ will be true, so $C \ \& \ D$ must be true by (1). Thus D , and hence $D \vee E$, is true, so G must be true by (2). This selection of truth values for $A, \dots, G$ satisfies (1–3) simultaneously.

c. Disjunctive normal form

- i. Existence of the form follows from consideration of truth tables.
- ii. You can also use various laws.
- iii. What is the disjunctive normal form of a contradiction?
- iv. From the disjunctive normal form you can see whether the conjunction of some sentences is satisfiable, and in fact tell under exactly which conditions. Try this for the previous exercise.
- v. If you have a disjunctive normal form for $\neg S$, then you can apply De Morgan repetitively to get a conjunctive normal form for S .