

1. *In class, the Cardinals II unit*
 - a. Ms. Sanford discussed *routine exercise 7* as follows.
 - i. Suppose $f: \mathbf{N} \rightarrow \mathbf{R}$. For each $i, j \in \mathbf{N}$ let f_{ij} be the $j+1$ st digit after the point in the decimal expansion of $f(i)$. Consider the real number $g = 0.g_0g_1g_2\dots$ where for each $i \in \mathbf{N}$, g_i is 0 or 1 according to whether $f_{ii} = 1$ or $f_{ii} \neq 1$. If g were equal to f_i for some $i \in \mathbf{N}$, then the $j+1$ st digit g_i after the point in the decimal expansion of g would equal f_{ii} , which is not the case. Thus $g \notin \text{Rng } f$, so f cannot be surjective. Therefore there is no bijection from $\mathbf{N}$ to $\mathbf{R}$. That is, $\mathbf{R}$ is uncountably infinite.
 - ii. Suppose $f: \mathbf{N} \rightarrow \mathbf{N}^{\mathbf{N}}$. For each $i, j \in \mathbf{N}$ let $f_{ij} = f_i(j)$. Consider the function $g \in \mathbf{N}^{\mathbf{N}}$ such that for each $i \in \mathbf{N}$, the value g_i is 0 or 1 according to whether $f_{ii} = 1$ or $f_{ii} \neq 1$. If g were equal to f_i for some $i \in \mathbf{N}$, then g_i would equal $f_i(i)$, which is not the case. Thus $g \notin \text{Rng } f$, so f cannot be surjective. Therefore there is no bijection from $\mathbf{N}$ to $\mathbf{N}^{\mathbf{N}}$. That is, $\mathbf{N}^{\mathbf{N}}$ is uncountably infinite.
 - iii. This argument, discovered by Cantor, is called the *diagonal argument*.
 - b. Discussion of a few points in the *Cardinals II* unit is postponed until later meetings.
2. *Measurable student outcomes*. I formulated these on the last page of the syllabus. I don't understand how they're to be used. But we've now covered outcomes 1 and 3–8. We haven't discussed outcome 2 because I haven't precisely specified ZF set theory. I'll do so later. We're now tackling 9–14.
3. *In class, the Boolean logic unit*
 - a. Here's what we are looking for, by Scott Kim: [click here](#).
 - b. One principle we'll stress is on a billboard on US101 near Santa Rosa, CA:
If we don't have it you don't need it! —Friedman's hardware
 - c. Check out Cooke 1969: "Rye whiskey" in contrapositive.
 - d. And here's another: [click here](#).
 - e. The relevant material in Stoll [1963] 1979 is Sections 4.1–4.5.
 - f. You'll struggle with proper usage of parentheses. Each text has its own conventions for the strength of the various operators. I won't be explicit about mine, lest you start believing that other texts adhere to it.
 - g. Here are selected exercises from Stoll. I used to assign these, but will consider them in class this year.
 - i. 4.2.7. Translate this sentence A into standard notation:

If either labor or management is stubborn, then the strike will be settled if and only if the government obtains an injunction but troops are not sent into the mills.

L = “labor is stubborn”

M = “management is stubborn”

S = “the strike will be settled”

G = “government obtains an injunction”

T = “troops are sent into the mills”

$A = “[L \vee M] \Rightarrow [S \leftrightarrow [G \& \neg T]]”$

Now suppose the following assumptions are accepted as true:

If the government obtains an injunction, then troops will be sent into the mills. If troops are sent into the mills, then the strike will not be settled. The strike will be settled. Management is stubborn.

(1) $G \Rightarrow T$

(2) $T \Rightarrow \neg S$

(3) S

(4) M

Determine whether, under these assumptions, A is true or false.

(5) Its antecedent is true, by (4).

(6) T is false, by (2, 3).

(7) G is false, by (1, 6).

(8) $G \& \neg T$ is false, by (7).

(9) The consequent $S \leftrightarrow [G \& \neg T]$ of A is false, by (3, 8).

(10) A is false.

Many writers have constructed wonderful examples of this kind of reasoning. One of the best was the mathematician Charles Dodgson, better known as Lewis Carroll.

- ii. 4.3.1.f. Translate this sentence from infix notation into prefix and postfix notation:

$[P \vee R] \Rightarrow [R \& [S \vee \neg P]]$.

Prefix: $\Rightarrow \vee PR \& R \vee S \neg P$

Postfix: $PR \vee RSP \neg \vee \& \Rightarrow$

This really didn't have anything to do with logic, but you should be familiar with this notation. Polish logicians adopted prefix notation in the 1920s, in part to analyze the complexity of logical statements without bothering about differing parenthesization standards, and in part to give a distinct memorable appearance to their work. In the 1950s, software engineers discovered how useful postfix notation is for programming translation of computer languages. They called it *reverse Polish*!

Define the notion, *fully parenthesized formula* (FPF) recursively as follows: first identify a set of variables (such as P, R, S), then stipulate

(1) all variables are FPFs;

(2) if Φ is an FPF and σ is a singular operator (such as $\neg$) then $\neg \Phi$ is an FPF;

- (3) if Φ, Ψ are FPFs and β is a binary operator (such as $\vee$) then $[\Phi \beta \Psi]$ is an FPF.

Recursive definitions of this sort are understood to confine the term defined to just those cases required by the rules.

The example infix formula of this exercise is not an FPF but it would be were it enclosed in parentheses. You can translate an FPF into prefix form by changing each $[$ to the corresponding β and omitting all occurrences of $]$.

Did you realize that you could omit about half of the parentheses you've been using without losing any information? Would the world be a better place? There's value in redundancy!