

1. *In class, the Axiom of Choice unit*
 - a. Reminder: the notation a_i is often used in place of $a(i)$ for the value of a function a corresponding to an argument i . For example, a sequence a that you might use in elementary calculus may be a function $a \in \mathbb{R}^{\mathbb{N}}$, with values $a_0, a_1, \dots$.
 - b. For a function $a \in X^{I \times J}$, we often use notation such as $\bigwedge_j a_{i,j}$ to stand for $\bigwedge \{a_{i,j} : j \in J\}$. The expression $\{a_{i,j} : j \in J\}$ then denotes the value of a function with domain I , so $\bigvee_i \bigwedge_j a_{i,j}$ stands for $\bigvee \{\bigwedge_j a_{i,j} : i \in I\}$.
 - c. *About assignment 14*
 - i. Routine exercise 3: think *small*.
 - ii. Routine exercise 4: this result suggests an alternative definition of $\preceq$. But if we used that instead of the one adopted in the *Cardinals I* unit, we would encounter questions much earlier concerning the axiom of choice.
 - iii. Routine exercise 5 prepares the way for a method, sometimes used in category theory, to discuss identity functions without referring to the elements of the corresponding sets.
 - iv. For most results involving forms of the axiom of choice, you wouldn't be able to find on your own the functions required until you gain some facility through experience. So on problems such as substantial 5, I give substantial hints. That makes it possible for you to gain some satisfaction from making something work out, while at the same time gaining that experience.
 - v. Substantial problem 4 is substantially the same as the step I mentioned earlier, where Prof. Ardila tacitly used the axiom of choice.
 - d. Substantial problem 1 shows that any family of functions f_i from a set X to sets A_i can be "simulated" by the projections $c \mapsto c_i$ from the direct product B of the sets A_i , and that there is a natural bijection between B and any other set B' that has this property. This allows direct products to be used in category theory, which speaks always of mappings between sets, never of the sets' elements.
 - e. Substantial problem 5 shows how to define the sum and product of an infinite family of cardinals. In parts 2, 3 there's a subtle use of the axiom of choice: from $\forall i [A_i \approx B_i]$ conclude $\forall i [F_i \neq \emptyset]$, where F_i is the set of all bijections from $A_i \rightarrow B_i$; by the axiom of choice there exists $c \in \prod_i F_i$. Then, for example, you can define a bijection $g : \bigcup_i A_i \rightarrow \bigcup_i B_i$ by setting $g = \bigcup_i c_i$. I suspect that the proposition that such a g always exists actually implies the axiom of choice, but I'm not at all sure.