

1. *In class, the Axiom of Choice (AC) unit*
 - a. I discussed the proofs that the AC implies Bernays's form and that Bernays's form implies the AC.
 - b. To prove the latter (substantial problem 2a), assume Bernays's form.
 - i. To prove: the AC.
 - ii. Given a family of nonempty sets A_i indexed by members i of a set I .
 - iii. To find: a choice function $c \in \prod_i A_i$.
 - iv. Consider the relation $R = \{ \langle i, a \rangle \in I \times \bigcup_i A_i : a \in A_i \}$.
 - v. Its domain is I .
 - vi. By Bernays's form, there is a function $c \subseteq R$ with the same domain I .
 - vii. This c satisfies the requirement of (iii).
 - c. I discussed the proof from the AC that surjections have right inverses.
 - d. Here's a proof that statement (c) actually implies the AC (substantial problem 2b). I'll assume that every surjection has a right inverse, and prove Bernays' form.
 - i. Given a relation R .
 - ii. To find: a function $c \subseteq R$ with the same domain.
 - iii. Consider the functions $l : R \rightarrow \text{Dom } R$ and $r : R \rightarrow \text{Rng } R$ such that $l(\langle x, y \rangle) = x$ and $r(\langle x, y \rangle) = y$.
 - iv. l is surjective, so by assumption has a right inverse $m : \text{Dom } R \rightarrow R$.
 - v. Thus $r \circ m : \text{Dom } R \rightarrow \text{Rng } R$.
 - vi. I'll show that $r \circ m$ satisfies the requirement for c in (ii).
 - vii. Given $\langle w, z \rangle \in r \circ m$.
 - viii. To prove: $\langle w, z \rangle \in R$.
 - ix. $\langle w, z \rangle \in m \mid r$ by (vii).
 - x. $\exists p [\langle w, p \rangle \in m \ \& \ \langle p, z \rangle \in r]$
 - xi. By (iv), $p \in R$, so $(\exists x, y) [p = \langle x, y \rangle]$.
 - xii. Now, $x = l(p) = l(m(w)) = w$ because m is a right inverse of l .
 - xiii. And $y = r(p) = r(m(w)) = z$ by (vii).
 - xiv. Thus $\langle w, z \rangle = \langle x, y \rangle \in R$ by (xi).