

1. *In class, Cardinals I.*
 - a. Ms. Crump discussed a proof of proposition 30.
 - i. She proved that X^Y is finite for finite sets X, Y by noting that it's a subset of $\mathcal{P}(Y \times X)$.
 - ii. I'd chosen to prove later that the power set of a finite set Z is finite by noting that it's equinumerous with $\{0, 1\}^Z$. That would be circular.
 - iii. But I noted that her recursive proof that $\#X^Y = m^n$ when $\#X, \#Y = m, n$ carries with it a proof that $\#X^Y$ is finite, so there's no problem.
 - b. I discussed a proof of proposition 31 and Cantor's theorem.
2. *History*
 - a. Cantor defined the sum and product of infinite cardinals in 1887–1888, sections VIII.5–7, and presented the commutative, associative and distributive laws. In [1895–1897] 1952, §3, he modified the definition of product, obtaining the one used in these notes. In §4 he defined exponentiation of infinite cardinals, and presented their arithmetic laws. For 1895 text, [click here](#).
 - b. Cantor published in [1891] 1996 the first version of his theorem that $\#\mathcal{P}X > \#X$, using the “diagonal” method of proof.
 - c. This result was regarded as potentially contradictory: if U is the universal set then $\mathcal{P}U \subseteq U$, which would imply $\#\mathcal{P}U \leq \#U$. Our system of set theory avoids that, since it admits no universal set.
3. There are many famous applications of the results in the *Cardinals I* unit, but those become somewhat mixed up with applications of the results in *Cardinals II*. The difference between these groups is very technical, so I'll postpone discussing them until *Cardinals II*.
4. *Project for a term paper.* There is some accessible literature about the evolution of Cantor's diagonal argument and its relation to the argument that leads to Russell's antinomy. A report on that might make a term paper.
5. *Project for a term paper.* Investigate extensions of the Cantor–Bernstein theorem. In particular, to get the theorem of Yente the Matchmaker in [Rosenholtz 2000](#) (temporarily online) is it merely necessary to require that the given injections be subsets of a given relation R and its converse, and to note that the constructed bijection is then included in R ? Is Rosenholtz's discussion of constructiveness equivalent to that in Sierpinski 1958, §2.6 exercise 2? Is Rosenholtz's title misleading? Does Yente's theorem occur in Tarski 1955 §3, Tarski [1928] 1930, Knaster and Tarski 1927, Tarski 1927, or Banach 1924? (Only very brief, easily translatable passages of those early papers should be relevant; I have copies of those that are not online.) These matters are discussed to some extent in Kanamori 1997 and in some detail in Wagon 1985, chapter 3. Finally, there are a number of “marriage theorems” in the combinatorics literature. To be sure, they're concerned only with finite sets, but are their proofs related to those considered in these other sources? I don't know the answers to any of these questions, but would like to.