

1. *Assignment 9.*
 - a. *Complete Lattices* routine exercises 3, 5.
 - b. *Complete Lattices* substantial problems 2, 3.
 - c. Tarski [1946] 1995, chapter 3, exercise 9.
 - d. Read Tarski [1946] 1995, chapter 4. In particular, be prepared to discuss its exercises 4, 7, 9, 10, 12, 17.
2. *Events*
 - a. The social discussed in the previous meeting will be at my house in the Sunset district, 6—about 8 PM, Wednesday 27 February. See y'all there!
3. *In class*
 - a. I responded to questions about Tarski [1946] 1995, chapter 3, in particular its exercises 1–3, 8, 10–12.
 - i. I was unsure why I'd emphasized exercises 1–3. Here's exercise 1.
 - (1) Given law III (symmetry of equality).
 - (2) Given law IV (transitivity of equality).
 - (3) Given the hypothesis of law V: $x = z$ & $y = z$.
 - (4) To prove the conclusion of law V: $x = y$.
 - (5) By (3), $y = z$.
 - (6) By (1), $z = y$.
 - (7) By (3), $x = z$.
 - (8) By (2), $x = z$ & $z = y \Rightarrow x = y$.
 - (9) By (6–8), $x = y$, q.e.d.
 - ii. The reason for doing this is that the various familiar rules about equality are interrelated. Studying that in detail would require extremely detailed reasoning of this sort, because the ideas are so close we don't normally distinguish them.
 - b. *Complete Lattices* unit
 - i. We discuss its routine exercise 4 and the last part of routine exercise 3, showing the five Hasse diagrams of five-element lattices (there are also five with fewer elements), those of the lattices of equivalence relations on sets with one to three elements, and those of the power sets of sets with one to three elements.
 - ii. We discussed of its routine exercise 2 further.
 - c. I discussed in detail the proof of Tarski's fixpoint theorem.
 - i. If x is any fixpoint of the function f in that theorem—i.e. $x = f(x)$ —then $x \leq x_0$ because x_0 is the supremum of all x for which $x \leq f(x)$. Thus x_0 is the *largest* fixpoint of f . A minor modification of the argument will yield a formula for the *smallest* fixpoint.

4. *History*

a. *History*

- b. The techniques introduced in the *Complete lattices* notes were in common use by 1890. The *closure* terminology comes from real analysis.
- c. The term *Moore family* is mine. It refers to Eliakim Hastings Moore. In the 1890s he founded the Mathematics Department at the University of Chicago, and the school of researchers known as the *American postulate theorists*. Later, he and others used these methods heavily in point-set topology.
- d. Click here for photographs of Moore, in [youth](#) and [maturity](#).
- e. During the 1920s Polish mathematicians, particularly Alfred Tarski and Casimir Kuratowski, pursued detailed applications of these methods in point-set topology, and introduced their use in logic.
- f. The fixpoint theorem marks Tarski's mathematical entrance in this course—his work was its most important inspiration. He thought of this argument during the 1920s. Its elegant publication in 1955 greatly simplified proofs in several areas of mathematics. It was presented as a model research paper in Math 880 this term. [Click here](#) for the relevant original text.
- g. Tarski was my Doktorgrossvater, and provided major inspiration for my recent work Marchisotto and Smith 2007 on the Italian geometer Mario Pieri, whose methodology Tarski adopted in the 1920s for his own geometrical work.
- h. [Click here](#) for photographs of Tarski in youth and maturity. Use password p&s .
- i. Feferman and Feferman 2004 is a fascinating biography of Tarski, from his struggles and conquests in Poland before 1939 and after then in the United States. At Berkeley in the 1940s he founded the world's leading school of logicians. The reviews Sinaceur 2007 and Wells 2007 are also enlightening.

5. *Projects*

- a. Moore's relationship to Germany, his founding of the Chicago department, the work of the American postulate theorists, and Moore's work in analysis would make a great survey for a paper in the history course.
- b. Pursue more details of the material in Tarski 1955. I've recently been finding related material, applied to computer science and economics, that is quite elementary and could contribute to such a project, but I'm not organized enough yet to report it here.