
1. *Events*

- a. We discussed plans for a “social” at my house some evening soon. The date will be announced in the next class outline.
- b. Saturday 1 March, all day. Annual meeting of the MAA Northern California and Nevada Section at CSU Sacramento, University Union, Ballroom I.
 - i. 1000–1100, Kevin McCurley, Google Research: Random walks on graphs
 - ii. 1100–1130, Student poster session
 - iii. 1130–1230, Susan Holmes, Stanford, Mathematics of flipping a coin
 - iv. 1230–1400, Lunch
 - v. 1400–1500, Jesus de Loera, Davis, Volume: hard to get, straightforward
 - vi. 1500–1600, Robert Lang, IEEE, The modern science of origamiSome of us from SFSU will be going, so a carpool might be feasible. But I’ll probably stay in Sacramento the night before, so won’t be available for that.
- c. Annual Tarski lectures in Berkeley. Each year a world-class logician gives a series of three lectures. This year’s are on the foundations of the theory of algorithms, by Iannis Moschovakis. He received his PhD from Kleene in Madison in 1963, has made major contributions to several areas, and is now retired from the University of Athens.
 - i. Monday 3 March 4 PM 60 Evans: Algorithms and implementations
 - ii. Wednesday 5 March 4 PM Room TBA: English as a programming language
 - iii. Friday 7 March 4 PM 60 Evans: Derivation of absolute lower bounds. (This collides with another such event at SFSU.)
- d. Fong Colloquium at SFSU. Each year a world-class gives a series of two talks here, the first for the scientifically literate public, the second for mathematics students and faculty. This year’s are on combinatorics, by Richard Stanley, Prof. Ardila’s MIT Doktorvater.
 - i. Thursday 6 March 4 PM Room TBA: Public lecture
 - ii. Friday 7 March 4 PM TH 211: Department lecture

2. *Homework*

- a. Student groups have agreed to be responsible for the following groups of *Cardinals I* propositions: 9–15, 18–19, 24–26. The other groups still need attention.
- b. I corrected a typo in *Equivalences and Partitions*, substantial problem 1: I changed R_n to R^n .
- c. I posted a solution of *Partially Ordered Sets*, substantial problem 1. There are 63 isomorphism classes of five-element partially ordered sets, of which 15 are self-dual. That makes 78 points; several of you scored in the 70s. The only common error I noticed was failure to recognize some examples as isomorphic. The point of the problem was to emphasize how hard that is. As

far as I know, there's no known formula for computing the number of isomorphism classes of n -element partially ordered sets.

- d. On the others, I've returned the following numbers of receipts for complete solutions:
 - i. Basic Set Theory routine 11, 15, 19. 3 each
 - ii. Assignment 2(d). 2
 - iii. Stoll 1.5.6. 3
 - iv. Tarski 1.6. 1
 - e. One of you has received four such receipts, one has received 3, and three have received two each.
 - f. Some of your solutions are quite good. One general comment for those who are in fact getting the hang of it: strive for neatness. Don't mention things that don't need to be mentioned. You'll just confuse yourself. It would also help me if you would say that you intentionally stopped short of a complete solution, so I don't go looking for things unnecessarily.
 - g. A few of you seem to lack some experience with abstractions commonly used in upper-division mathematics. In particular, you confuse elements of sets and subsets of sets, particularly in the context of power sets, unions, and intersections. And when you need to prove $A \subseteq B$, you confuse the step, "let t be an element of A ; I must prove t belongs to B ," with "assume A is not empty, then, ..." Perhaps the prevalence of empty-set considerations in the *Basic Set Theory* trivial questions has made you overconscious of them. But those were intended mainly to force precise thinking and to emphasize that counterexamples to false elementary statements are often the simplest conceivable.
 - h. Another failing is harder to describe. A common technique is to introduce an object such as t in the previous item, prove it has some property $P(t)$, then later, observe that since t was subjected to no conditions, the statement $\forall t P(t)$ is true, and can be used as a hypothesis for some later argument. That step is called *universal generalization*. Another technique: after a step of the form $\exists u P(u)$, usually an assumption or the conclusion of a theorem cited from elsewhere, you introduce u as some object for which $P(u)$ is true, then use it in further steps. That step is called *existential instantiation*. It's vital to keep track of how t and u were introduced, because t represents an arbitrary object, while u is specific. You can't universally generalize with u . This means you need to do steps like these in proper order, and even repeating your words later for emphasis is possibly wrong, because it would affect that order. Tarski [1946] 1995 speaks to such issues, but probably not very effectively. You're expected to adopt good habits by observing proofs in texts and classrooms.
3. In class I discussed the *Complete Lattice* unit's routine exercise 1, to explain a subtle difference in terminology between that used for complete lattices in general and that used for the real number system in particular.

- a. By definition, a complete lattice is a partially ordered set $\langle X, \leq \rangle$ for which each subset $S \subseteq X$ has an infimum $\wedge S$ and a supremum $\vee S$ in X . In particular, X has minimum and maximum elements $\wedge X$ and $\vee X$.
- b. You learn that the real numbers form a partially ordered set $\langle \mathbf{R}, \leq \rangle$, and every *bounded* subset $S \subseteq \mathbf{R}$ has an infimum and supremum in $\mathbf{R}$. But *unbounded* subsets do not: the subset $\mathbf{N} \subseteq \mathbf{R}$ has infimum 0 but no supremum. Thus $\langle \mathbf{R}, \leq \rangle$ is not a complete lattice. But it *is* called a *complete ordered field*. That term can be defined several ways, which are equivalent to being an ordered field each bounded subset of which has an infimum and a supremum.
- c. In many real analysis classes and texts, the partially ordered set $\langle \mathbf{R}, \leq \rangle$ is augmented by two new elements to get the *extended* real number system $\langle \mathbf{R}^*, \leq^* \rangle$, where $\mathbf{R}^* = \mathbf{R} \cup \{-\infty, \infty\}$ and $\leq^*$ is the same as $\leq$, except that $-\infty$ and ∞ are new minimum and maximum elements. With that definition, it is shown that *every* subset of $\mathbf{R}^*$ has an infimum and supremum in $\mathbf{R}^*$.
- d. Routine exercise 1 shows that this construction works with *any* partially ordered set $\langle X, \leq \rangle$ for which each bounded subset $S \subseteq X$ has an infimum and a supremum in X : the corresponding partially ordered set $\langle X^*, \leq \rangle$ is a complete lattice.