

1. *Assignment 5.*
 - a. Tarski [1946] 1995, chapter 1, exercise 6.
 - b. *Partially Ordered Sets* routine exercise 3.
 - c. *Partially Ordered Sets* substantial problem 1, with a *deadline*: start of the meeting after next. I won't recirculate that problem for correction.
 - d. *Partially Ordered Sets* substantial problem 4.
 - e. Read the *Complete Lattices* unit. In particular, prepare to discuss routine exercises 2, 4, 6. I'll cover this material in lectures 6, 7. Locate and read related material in Stoll [1963] 1979.
2. I responded to questions about Tarski [1946] 1995, through chapter 1.
 - a. When quantifiers range over a finite set $x_1, \dots, x_n$, quantified clauses can be replaced by (usually overlong) disjunctions or conjunctions:
 - i. $\forall x [P(x)] \Leftrightarrow [P(x_1) \& \dots \& P(x_n)]$
 - ii. $\exists x [P(x)] \Leftrightarrow [P(x_1) \vee \dots \vee P(x_n)]$.
 - b. With an infinite range, however, that can't be done because we can't utter infinitely long sentences. However, some parts of logical theory work even with infinitely long unspeakable conjunctions and disjunctions. The study of such considerations is called *infinitary logic*.
 - c. There are several notations in use. In more recent years, Tarski used $\bigvee$ for $\exists$ to emphasize its connection with $\bigvee$. (The latter stands for the Latin *vel*, which means *or* in the inclusive sense.) Tarski also used $\bigwedge$ and $\bigwedge$ for $\forall$ and $\&$.
3. I responded to questions about the *Partially Ordered Sets* unit.
 - a. We discussed Hasse diagrams and substantial problem 1, which is assigned to be collected at the meeting after next. There are substantially more than fifty isomorphism classes of five-element partially ordered sets.
 - b. Some students are confused because they've only encountered special Hasse diagrams of the power sets of some small sets, not the general idea. Don't apply special considerations to the general case.
 - c. I stressed the difference between *maximal* and *maximum*, *minimal* and *minimum*.
4. *History*
 - a. The theory of partially ordered sets stems from the 1930s, but most of the material in the *Partially Ordered Sets* unit was in common use by 1890. Garrett Birkhoff, son of the Harvard professor and foremost American mathematician George David Birkhoff, assembled existing material and began contributing new results during his term as Harvard Junior Fellow after his BA degree there in 1932. He wrote the "bible" of the subject, Birkhoff 1948, originally published in 1940. [Click here](#) for a 1928 photo.
 - b. In a wonderful Spring 1962 Math 890 course at SFSU based on that text, Alden Pixley really introduced me to research mathematics.

- c. Helmut Hasse was a leading 20th-century algebraist. I don't know why those diagrams are named for him.
- 5. *Projects for this course*
 - a. Some information on the project described in the *Partially Ordered Sets* unit is recorded in the cited literature. Bjarni Jónsson has published some papers on that subject.
 - b. Describe and implement the algorithm in Flanders 1987 for generating the transitive closure of a Boolean matrix. Is it useful for counting isomorphism classes of four-, five- and six-element partially ordered sets?
- 6. *Projects for master's expository papers*
 - a. Report progress on the question, how many isomorphism classes of finite n -element partially ordered sets are there. I have some references.
 - b. I have many reprints of or references to papers, mostly in English, that pursue by elementary methods other easily understandable questions about partially ordered sets. I suspect that some could lead to master's projects in combinatorics or algebra. Galvin 1994 seems particularly promising.