

1. *Assignment 2*
 - a. [Basic Set Theory](#) routine exercises 11, 15, 19.
 - b. [Basic Set Theory](#) substantial problems 2, 3.
 - c. Stoll [1963] 1979, exercise 1.5.6: find a *neat* solution, expressed mostly in English.
 - d. Between $\bigcup_{i \in I \cap J} A_i$ and $(\bigcup_{i \in I} A_i) \cap (\bigcup_{i \in J} A_i)$ should stand either $\subseteq$ or $\supseteq$. Decide which one makes a theorem, and prove it. Find a counterexample (with *tiny* I, J, A_i) to show that the other one doesn't. Continue the discussion for other statements with some or all of $\cap, \cup$ replaced by their dual operations.
 - e. Read Tarski [1946] 1995, prefaces. Read Church's JSL reviews of the book.
 - f. Read [The Need for a Rigorous Set Theory](#). Prepare to discuss this during the third class meeting. Locate and read related material in Stoll [1963] 1979.
 - g. Read [Equivalences and Partitions](#). Prepare to discuss this during the third class meeting—in particular, routine exercise 2. Locate and read related material in Stoll [1963] 1979.
2. *References*
 - a. Rather polished sets of notes, such as those mentioned in assignment 2, I call *units*.
 - b. Units generally contain references to textbooks, monographs, or expository papers that approach the subject in much the same style. Often those show where I learned the approach. I do not attempt to list there more recent publications that might contain equivalent or better presentations. But I may mention such publications in class outlines, and encourage you to mention any that you find.
 - c. The units don't cite sources of results covered in most texts. But they do cite those of some particularly noteworthy definitions, arguments, and theorems.
 - d. Class outlines will often include references to related material not discussed in the units.
 - e. All such citations refer to the course bibliography for detailed publication information.
3. *In class*
 - a. I responded to questions about the *Basic Set Theory* unit.
 - i. One involved the *second principle of recursive definition*. I'll respond in detail in outline 3.
 - b. We discussed some of routine exercises 1–10, 15b, 18.
 - i. Exercise 1 in detail, with a direct proof and one by contraposition.
 - ii. The organization of a solution of exercise 3 is necessarily case-ridden and messy. That proof is unlike anything else we'll be doing for a while. I'll return to it later.

- iii. In parsing exercise 4, note that my $\Leftrightarrow$ connective is stronger than the $\vee$ connectives: the latter are to be performed first.
- iv. Exercise 5f solution: if $X \neq \phi$ then $\cap X' \subseteq \cap X$. The proviso is required because if the empty set had an intersection, the latter would have to be a universal set, and there isn't any universal set.
- v. We may return to some of these during the next meeting.
- c. I discussed *assignment 2* briefly.
 - i. I stressed the engineering **KISS** principle: **Keep It Simple, Stupid!** This should apply to all quests for counterexamples, and for many proofs. If an argument appears overcomplicated, it probably is. I'll emphasize this again and again.
 - ii. In exercise 19 it might help you psychologically to use the letter G instead of the traditional notation $\tilde{F}$.
- d. I forgot to ask for responses to the collective nouns exercise—we may laugh about them next time!
- e. *Without definition*: this phrase occurs early in the *Basic Set Theory* unit. We can't define *everything*, because we have to start somewhere. Anyway, these notes describe existing practice. They are not intended to provide a foundation.
- f. *Equality for objects that aren't sets* must be considered separately. Analogously, when you define a new type of object in object-oriented programming, you should first write a procedure for telling when two objects of that type are to be regarded as equal.
- g. The term *power set* stems from material in the notes *Cardinals I*.
- h. If we used *suffix* instead of *prefix* notation for functions, we wouldn't need to distinguish relative product from function composition.
- i. Order of composition is important! [This cartoon](#) brings to mind Russell's quip, that the equation $(G \circ F)^{-1} = F^{-1} \circ G^{-1}$ corresponds to our donning socks before shoes, then having to doff shoes before socks! Programmers call it the *LIFO* principle: *Last In First Out*.
- 4. *History*
 - a. Most of the methods described in these notes before the section on natural numbers were in common use by about 1890, with varying terminology. Richard Dedekind and Giuseppe Peano published works around then that dwelt on these methods. Previously, they were introduced as necessary for other exposition, and rarely discussed in detail.
 - b. But some distinctions familiar today didn't become common until around 1900: for example, those between an object and its singleton, and between a relation and its converse.
- 5. [Schwenk 2002](#) illustrates the practicality of distinguishing $\phi, \{\phi\}, \{\{\phi\}\}$.
- 6. *Project*. Category theory is a fundamental subject in advanced algebra. Basing it on the set theory presented in this course requires limiting its strength inconveniently. Solomon Feferman recently presented a colloquium at Berkeley on strengthening set theory to accommodate category theory. That involves providing

- some features analogous to those of a “universal set” while avoiding contradictions analogous to Russell’s. For someone already quite familiar with category theory, an investigation of this problem area and current research could result in a term paper. A more comprehensive study could lead to a master’s expository paper.
7. *Project* for a term paper. The theory presented informally in the *Basic Set Theory* unit will be incorporated into Zermelo–Fraenkel set theory (ZF) later, after the formal syntax of first-order logic is introduced. Some features of this theory may seem out of step with logical methods you learned in elementary courses—particularly its insistence that no set should be “universal,” containing all objects. Many texts employ that notion in developing simple Boolean logic. These notes don’t, because ZF avoids it to forestall Russell’s antinomy. Logicians have investigated alternative formal set theories that do include a universal set: they employ different means to avoid the antinomy. Recently, interest has increased in one of them, called NFU. The unpublished manuscript Holmes [1998] 2007 contains a presentation of NFU analogous to this course’s exposition of ZF. A thorough comparison of the elementary parts of the two theories could constitute an excellent term paper. It would be a good choice for someone who has already studied elementary logic.