

**MATH 800 FOUNDATIONS OF MATHEMATICS
OUTLINE 1**

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SPRING 2008**

1. I'll post outlines such as this at first with the notation "Tentative" on the title line. I'll revise them after they're covered in class, and remove that notation. After that, I'll won't change them unless I notify you explicitly. Don't trust hardcopies made before those revisions!
2. *Assignment 1*
 - a. Investigate the course [Syllabus](#).
 - b. Turn in to me ASAP a sheet of paper with
 - i. Your e-mail address, if it should differ from the *.sfsu.edu one that I have.
 - ii. A list of relevant courses you're now enrolled in (I already have your transcript).
 - iii. Any other comments about subjects related to this course, that you might want to consider for a paper topic.
 - c. Investigate the course [Bibliography](#).
 - d. Read Kneebone's JSL review of Stoll [1963] 1979.
 - e. Investigate [Typefaces I Use in Mathematics](#).
 - f. Read the [Basic Set Theory](#) notes.
 - i. You may want to locate and read related material in Stoll [1963] 1979 as referred to in the notes.
 - ii. Prepare to ask questions about them during the second class meeting. In particular, prepare to discuss its routine exercises 1–10, 15b, 18.
 - iii. Hand in answers to its *trivial questions* at the beginning of the *third* class meeting.
 - g. Think about this—we'll laugh about it during the next two class meetings:

A *collective* noun refers to a collection of things or concepts: for example, *collection* is a collective noun. English has a surprising number of collectives. Some refer to collections with a specific *number* of elements: for example, a *triple*. Some refer to collections of a specific *kind* of elements: for example, a *gaggle* of geese. For each letter of the alphabet find a collective noun beginning with that letter that does *not* imply a specific number of elements. For the letter *x*, you may select a noun beginning with *ex*.
3. *Later homework assignments.* Numerous homework problems will be required, many of which are quite challenging. I'll assign them as soon as the appropriate background is covered in class.
 - a. Hand in solutions ASAP, even if incomplete. Hand in solutions to *separate problems separately* unless otherwise instructed.
 - b. I'll keep complete solutions and give you receipts.
 - c. I'll usually criticize partially correct solutions and return them for your continued work, repeating this until the solution is complete or the semester

- ends. You must *keep all work on a problem together*: that is how I determine how near you are to a complete solution.
- d. At semester's end you'll submit all remaining partially correct solutions for partial credit.
 4. *Set Theory*. Because Math 800 is a graduate course and SFSU doesn't regularly offer the undergraduate course that most departments would regard as its prerequisite, part 1 of Math 800 will be devoted to a streamlined but sophisticated survey of just those parts of elementary set theory that are essential to understanding the material on logic that will follow. The approach will seem very algebraic to you.
 - a. We'll consider the axiom of choice and cardinal arithmetic in detail, but ordinal arithmetic hardly at all.
 - b. Part 1 of Math 800 is supported by very detailed notes posted on my website. It is also covered in a somewhat less focused way in Stoll [1963] 1979. You are responsible for locating and reading related material in that source.
 5. *Basic Set Theory*. These notes are meant for various undergraduate courses as well as this one. In fact, I've included some of this material in courses on proof and exploration in mathematics, modern algebra, discrete mathematics, and set theory, and would refer to it should I ever teach real analysis. Therefore I kept these notes spare. In particular, I've hardly mentioned the history of this material. I'll add a little here. More information will come in later outlines.
 - a. The documented source of Bertrand Russell's antinomy is his [1902] 1970 letter to Gottlob Frege.
 - b. Russell's letter appears in Heijenoort [1967] 1970, a wonderful source of historical lore on this course's material. The editor's discussions of its included papers are particularly useful.
 - c. The documented source of Casimir Kuratowski's definition of ordered pair is Kuratowski 1921, 171.
 6. *Projects*. You may be searching for topics appropriate for a term paper in this or another course or a master's expository paper. All mathematics professors should be able to suggest topics in their areas of greatest competency. For me, those are foundations of geometry, and some parts of logic, numerical analysis, software engineering, and history of mathematics. As possible projects occur to me during this course, I'll note them in these outlines. In almost every case, further conversations would be required to clarify the scope and feasibility of a possible project. Please don't think that you need to select a project from *this* cumulative list. It is intended, rather, to suggest the *sort* of project that would be appropriate.
 - a. *Project* for a term paper. Describe and discuss some alternatives to Kuratowski's definition of ordered pair. I think a very young Norbert Wiener proposed one a few years before Kuratowski. Was it the first? How were ordered pairs handled before such a definition? Harris 1970 considers other alternatives, and points to another paper.
 - b. *Project*, but not for *this* course. The terms "injection", "surjection", and "bijection" stem in part from Franco-German rivalry in mathematics and other areas, and thus open the door to a general topic: that rivalry.

- c. *Project*, maybe for this course. Kuratowski's work stems from Polish nationalism, specifically in mathematics, and thus opens the door to that topic. Some background literature is in English. Polish mathematical research was often published in French, and concentrated heavily on foundations of mathematics. Polish itself is probably not required for this project.
- d. *Project* for a term paper. Compare and contrast, in critical detail, at least two methods of constructing the real numbers from the rationals or the natural numbers and proving that they form a complete ordered field. Many textbooks cover a single method in detail; a paper should be significantly broader than that, but might skip some details, as long as it says what is skipped. I don't think I've seen Weierstrass's method in any textbook; but enough information may be available to include it in a project.