

1. This class period lasted from 10:45 to 13:15, Thursday, May 21, in TH428.
2. *First term papers*
 - a. The second set of term papers came out very well. I was particularly pleased that twelve students' second-paper grades were more than one point higher than their first-paper grades. *Congratulations!* (The maximum scores were both 19. Eleven students' grades varied one point or less, up or down. Four went down more than one point.) Here is the distribution:

i.	80 ... 100%	15.2 ... 19.0	xxxxxx xxxxxx x
ii.	65 ... 80%	12.4 ... 15.1	xxxxxx xx
iii.	50 ... 65%	9.5 ... 12.3	xxxxxx xx
iv.	40 ... 50%	7.6 ... 9.4	xx
v.	0 ... 40%	... 7.5	
 - b. Here are the titles of the top eight papers:
 - i. Math and War: How Mathematics Led to America's Rapid Technological Advancement during World War II
 - ii. John von Neumann and Ideas to Potentially Jiggle the Planet
 - iii. Znam's Problem: A Number Theory Wonder
 - iv. Artists of Islamic Geometric Patterns
 - v. Tropical Mathematics and the Fundamental Theorem of Algebra
 - vi. The Period of *Wasan*
 - vii. Finding Their Own Way: Early-Age Thought Process on the Rainbow
 - viii. A Conflicted Calculus: The Story of Newton and Leibniz
3. *Quiz 4*
 - a. covered the material mentioned in lectures 21, April 21, through today's, (but not material in student reports). The corresponding lecture files indicate which sections of Struik and Kennedy were covered. The structure of the quiz was similar to that of the previous ones.
 - b. It will be debriefed via individual emails.
 - c. The results were bad, in strong contrast to the second term papers. Evidently students gave term-paper work marked precedence over preparation for the quiz. That paid off for the term papers, which were more important. But, should I ever offer this course again, I would insert a unit on studying for a history course.
4. Ms Brown reported on the Bernoulli family. Alongside their famous mathematical achievements, she emphasized family turmoil. The family originated in Antwerp, now in Belgium. Victimized by prejudice against their Calvinist religion, they fled to Basel, Switzerland, but evidently retained personal as well as academic connections with the Low Countries. They dealt with each other deplorably. It surprises me that there is no readily available book-length biographical study of the Ber-

- noullis. I'd guess that there are many archival materials available about them in Basel and elsewhere. If so, there's a wonderful PhD history project!
5. Mr. Yang reported on Japanese mathematics, particularly that known as *Wasan*, which developed during the period 1639–1868 when Japan closed itself from contact with the outside world. Mathematics had developed there previously, and had been imported to some extent from China and by the Jesuits; the resulting discipline progressed, slowly, during the time of closure. Mr. Yang mentioned some developments in what we would call approximation theory, toward computing π with high accuracy. Most spectacular, though, is the tradition of *sangaku*, which was evidently not well-known even in Japan until recently. This consisted of intricate Euclidean geometry problems posted in temples as offerings and suggestions for contemplation. Fortunately, they are accessible now in both Japanese and English. After the reopening, Japanese mathematics integrated very rapidly into the world-wide mathematics community.
 6. Ms Lee reported on the development of perspective drawing during the Renaissance, particularly in Northern Italy. Various scholars, talented in both art and mathematics, discovered basic principles, extending rudimentary work found even in Euclid. These included Alberti, Brunelleschi, Pacioli, Leonardo da Vinci, Dürer, and others. Soon some of them published manuals to guide artists, and the techniques spread slowly throughout Northern Europe. (It is very interesting to look at paintings from that period displayed in museums, to see how their sophistication depended both on the artist's ingenuity and on his contact with this influence.) Later, in France, Desargues published material on drawing techniques that eventually led to major theorems in projective geometry. Brook Taylor, an English contemporary of Newton known for Taylor series, published a major work on perspective that introduced some of today's terminology and methods.
 7. Ms Dang reported on the development of hyperbolic geometry. This subject originated with attempts to prove the parallel postulate from the remaining Euclidean postulates. Analysis of attempts by Saccheri, Lambert, Legendre, and others revealed that the authors had left unstated other assumptions, such as the sum of the angles in a triangle's being 180° . They had merely proved that those assumptions were equivalent to the parallel postulate, just as we found in class that Playfair's postulate is. In the early 1800s, J. Bolyai, N. Lobachevski, and Gauss all developed enough consequences of a *negation* of the parallel postulate—that there should be *more than one parallel* to a given line through a given point not on it—that it *seemed* that the negation, together with the other Euclidean postulates, entailed a consistent theory, *hyperbolic geometry*. Gauss didn't publish, and the others' work was published only obscurely, hence the subject found little attention for decades. In the mid- to late-1800s, E. Beltrami and F. Klein found that hyperbolic geometry actually precisely describes some mathematical structures, overlooked until then, and is hence a consistent theory. Interest in it then increased markedly, and it now has applications in many other areas of mathematics. (On my advice, Ms Dang omitted all mention of the consequences of another negation of the parallel postulate, that there should be *no parallel at all* to a given line

- through a given point not on it. That leads to another alternative theory, often called *elliptic* geometry, or, less accurately, *Riemannian* geometry.)
8. Mr. Long reported on the development of trigonometry. A few of the propositions in Euclid's *Elements* are really basic trigonometry. Later work extended them to handle triangle calculations required for astronomy. This included spherical trigonometry, and the law of sines in both spherical and plane forms. This development occurred in Alexandria, India, and the Middle East. At the same time, mathematicians in all these areas discovered how to compute accurate tables of trigonometric functions. Their methods involved use of trigonometric formulas as well as approximation techniques now regarded as part of calculus. During the Renaissance, some of this work spread to Germany. The present-day presentation of trigonometry, and its relation to calculus, is largely the 1700s work of Euler.
 9. Mr. Lee reported on the influence of the mathematician Georg Pólya. Born in Budapest in a Jewish family converted to Christianity, Pólya was educated there. He only slowly came to emphasize mathematics as his calling. As a junior mathematics professor in Zürich, Pólya started with his friend Gábor Szegő a series of problem-oriented books in analysis, now available in English. A totally new approach, these guided many students' and scholars' studies of higher mathematics for years, including my own. In collaboration with Swiss crystallographers, he applied modern algebra techniques to classify repetitive designs; this provided the foundation for analysis of many types of art. He contributed many major results in analysis, probability theory, and combinatorics. Pólya and Szegő emigrated to the U. S. to escape European oppression, and settled in Stanford. There, Szegő became chairman, and Pólya lent more emphasis to problem-solving texts. Several of those have become classics in undergraduate mathematics education. Even after retirement Pólya continued close contact with local high-school students and teachers. He was a beloved figure of our community until he died at age 98, about twenty-five years ago.
 10. Mr. Tejada submitted an abstract of his paper on the inadequate treatment of non-European, particularly Islamic, history of mathematics, in our curriculum and literature. I tend to agree with his views, particularly concerning elementary mathematics. That is, if this course had been structured to emphasize only the mathematics covered through elementary calculus, but otherwise unchanged, the non-European sources would have been sorely underemphasized. Part of that is due to my choice of Struik as text, chosen for its conciseness, clarity, revolutionary role, and price. (And part is due to my not having studied that material.) During the 1940s Struik was in fact a pioneer in bringing attention to non-European sources. Much has occurred since, but that still is not found in every such text. To some extent that is to be expected since the non-European sources are still undergoing fundamental historical research. But even that work is not yet adequate disseminated. We hope that situation will improve, at an accelerated rate.