

1. *Quiz 3* was debriefed via hardcopy.
2. Ms. Zeldow presented a brief report on the type of research she did for her first term paper, a history of the development of the theory of Ehrhart polynomials. These are used to count the number of points with integer coordinates inside and on a polygon, polyhedron, or higher-dimensional polytope whose vertices are integers. Eugène Ehrhart discovered them about fifty years ago and wrote many papers on the in the proceedings of the French Academy of Sciences. A high-school teacher who was awarded the doctorate only at age sixty! Since he worked outside the usual framework of mathematical research, it was difficult to find secondary sources about his biography and research. (It might have been easier to find such material in journals addressed to French high-school teachers, but time and library resources didn't permit that.) So Ms. Zeldow had to rely on some of his original papers, and on various means of translation.
3. Ms. Velasquez presented a brief report on the type of research she did for her first term paper, a study of the mathematician Alfred Lotka, partially responsible for the Volterra-Lotka predator-prey equations considered in most courses on differential equations. Lotka contributed much energy and many results to the fields of mathematical biology and population statistics. He too, worked outside the familiar academic context, for various government agencies, research organizations, and publishers. She found biographical materials of various sorts, and relied to a major degree on prefaces that he wrote for his own publications. One most interesting aspect of these was his acknowledged debt to the spirit of Wilhelm Ostwald, a Nobel-prize-winning German chemist who conveyed philosophical principles akin to those of the logical positivist movement in the early 1900s.
4. *Struik*, 7.6–7.8, continued
 - a. Note the collection of luminaries that Frederick the Great attracted: earlier Euler, now Maupertuis and Voltaire. I was delighted once to see Voltaire's quarters in the Palace Sans-Souci in Potsdam, Frederick's summer retreat. Wonderfully airy and not very large, it was designed as an imitation of French frillery. [Click here](#) for my 2007 photo.
 - b. Notice that by the mid-1700s French mathematics was written in French. And the British wrote English.
 - c. Struik noted that with the study of solutions of differential equations in the mid 1700s the question arose about how general is the notion of a function. Must solutions always have algebraic expressions? Are infinite series, non-elementary integrals, improper integrals, etc., to be admitted as possible definitions of functions? Can useful functions be discontinuous? Today these questions receive obvious "yes" answers. Back then intoxication with the success of algebraic methods was so prevalent that mathematicians were reluctant to admit there's anything beyond that. They didn't have to for at least another century.

- d. Struik returned again to the achievements of the French encyclopedists. He noted that the first books on history of mathematics arose from that group.
- 5. *Struik 7.9, on Lagrange*
 - a. Joseph-Louis Lagrange made his entrance in midcentury. You may have thought he was French, but his real name was Luigi Lagrangia, and his native language was the Piedmontese dialect of Italian. Ruled by the Savoy dynasty, Piedmont was a buffer (and buffeted) state between France and Austria. Lagrange's first employment, in 1755, was with the military school in the capital, Turin, where Peano and Pieri would teach 150 years later. When they were born, the Savoy realm was called the Kingdom of Sardinia.)
 - b. [Click here](#) for a picture of his statue in the Academy of Sciences in Turin.
 - c. But that was not the center of things scientific, so in 1766 Lagrange accepted Frederick the Great's invitation to succeed Euler at the Berlin academy. On Frederick's death in 1786 Lagrange leapt into the melée of pre-revolutionary France. Evidently quite diplomatic, he survived the bloody revolution unscathed. He spent his last years at l'École Normale Supérieure and l'École Polytechnique, which I discussed in lecture 21.
 - d. Lagrange and his French contemporary Laplace were both involved in the formulation of the metric system of measures.
 - e. Lagrange introduced a new algebraic flavor into calculus. By assuming that all functions occurring in his study were representable by power series, he could derive the familiar results of calculus without using infinitesimals, by purely algebraic procedures applied to the series' coefficients. He developed a tremendous number of applications of calculus, even to areas of mechanics previously regarded as mainly geometrical, without the continual recourse to geometric arguments previously thought necessary.
 - f. But of course he couldn't really justify his fundamental assumption. The question remained whether functions could or should be studied which are not representable by power series.
 - g. Lagrange also did major work in the theory of numbers: for example, the theorem that every natural number is the sum of four squares.
 - h. And he developed the theory of polynomial equations of degree higher than four, introducing permutations of the roots, preparing the way for the negative results of the 19th century. The permutations form groups and subgroups: one of the early theorems today in a group-theory course is Lagrange's.
- 6. *Miscellany*
 - a. *Some words*
 - i. *Geometer* used to mean any mathematician, or particularly one who pays strict attention to deduction. Pascal's *The spirit of the geometer*, for example. This term was still used well into the 1800s.
 - ii. *Analysis* used to mean—until around 1900—mathematics in general.
 - (1) *Infinitesimal* analysis meant what we call analysis today: mathematics using the methods of calculus.

- (2) *Algebraic* analysis meant what we call algebra today.
- b. *Universities*: We've seen the center of mathematical research shifting from academies of science to universities.
 - c. *Journals, specialization*
 - i. Similarly, publishing of mathematical research began to broaden. Journals dedicated to mathematics were founded to supplement the proceedings of the academy meetings.
 - ii. The French mathematician Gergonne (not associated directly with the Paris establishment) founded in 1810 the first journal exclusively devoted to mathematics: *Annales* (later *Journal*) *de mathématiques pures et appliquées*. It is still published.
 - iii. August Crelle, the mathematician instrumental in developing mathematics and science at the Prussian universities, founded in 1826 the first German mathematics periodical, *Journal für die reine und angewandte Mathematik*, which is still published. Its name imitates that of Gergonne's. It just came online at the Göttinger Digitalisierungszentrum, so you can look at it. First, [click here](#) for a list of online mathematics texts maintained by the University of Bielefeld, in Germany. Then go to titles beginning with J, and follow your nose. (Somehow this one escaped the SFSU list.)
 - iv. Although both gave pure and applied mathematics equal billing (*pures* or *reine* versus *appliquées* or *angewandte*) both eventually became bastions of pure mathematics. I suspect but do not know that mathematics research faculties in both countries, like those in this country today, touted parity as well—the better to argue for funding—but in practice gave by far the greatest emphasis and prestige to pure mathematics. I wish I were familiar with historical research on this question, and of the politics of the German university systems during the 1800s, simultaneous with industrialization and political unification. The only works I've noticed on that subject were of East German origin some decades ago, and seemed too heavy for me. Of course, the commonplace argument today is that most advances in applied mathematics stem from earlier work in pure mathematics. Personally, I feel that our discipline would benefit from erasure of the distinction between pure and applied subdisciplines.
 - v. Just as journals specialized in mathematics, from about 1800 on, mathematicians began to specialize in *parts* of mathematics.
 - vi. Exceptions—big ones! *Gauss*, *Legendre*, and *Cauchy*.
 - (1) Struik noted that Gauss and Legendre they covered about the same areas, but Gauss went deeper into everything. By 1870 or so, Gauss was recognized as the most important mathematician of 1800–1850.
 - (2) Gauss was employed by the Observatory at the University of Göttingen, and that in fact took a lot of his time. He taught little,

and had few research supervisees. Legendre taught at the two Écoles in Paris. Cauchy's career was more complicated due to political factors.