

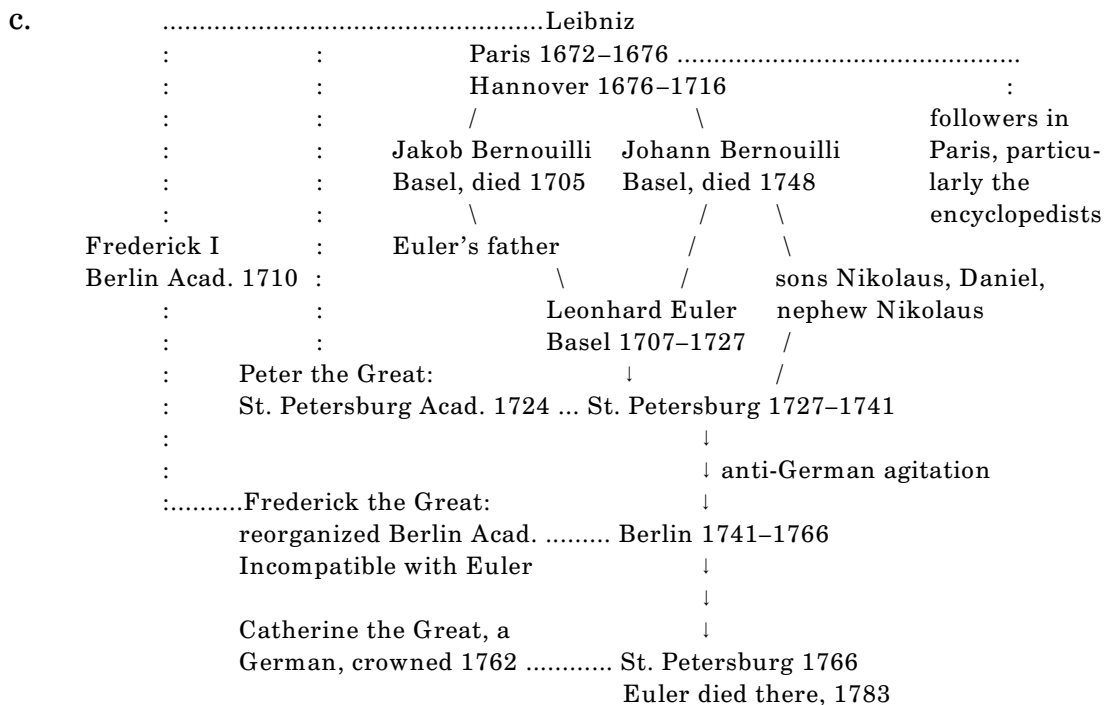
1. *Assignment:* resume reading Kennedy [1980] 2006. I will supplement it with discussions of aspects of Peano's cultural milieu that are probably unfamiliar to you.
2. *Quiz 2* will cover material through the end of Struik, section 7.5, and this lecture.
3. *Struik, sections 6.7–6.8*
 - a. Newton's approach to calculus emphasized motion; Leibniz's, static geometry.
 - b. Leibniz' notation and terminology was more suggestive and convenient, and became ours.
 - c. Leibniz introduced the general techniques for solving max-min problems, finding inflections, etc., that we use now.
 - d. Struik mentioned briefly Newton's method of approximating a root r of a function f . I suppose he meant what we call Newton–Raphson iteration: make an initial estimate x_0 to r , then apply the formula $x_{n+1} = x_n - f(x_n)/f'(x_n)$ recursively to generate a sequence of approximations x_n that approach r . This process works if f satisfies certain differentiability requirements. It underlies most algorithms for this problem used today. (Raphson was Newton's colleague.)
 - e. Struik mentioned briefly Newton's classification of the cubics. One way to imagine it is to remember how a conic can be generated as the intersection of the plane $z = 1$ and a cone with apex at the origin, and how the conic varies as you rotate the cone around its apex. The equation of the cone (in 3D) is quadratic. Is there a single type of surface with a cubic equation that you can rotate about the origin so that its sections with that plane form all (or most of) the cubic curves? I think not one, but several, which underlie the classification, but I don't know the details. The algebra is serious!
 - f. Struik *neglected* to mention the major extent to which Newton, in later life, was preoccupied with alchemy and arcane religious studies. The recent TV documentary, on the other hand, seemed to sensationalize them somewhat.
 - g. During the next century followers of Leibniz and Newton argued mightily about which of the two had been first with calculus. This was fruitless: it's clear now that the two worked independently. But the dispute created a barrier between England and the Continent, and English mathematics stagnated as a result.
 - h. Want to entertain a calculus class? Have them decide which is tastier, fig Newtons by National Biscuit Company or Leibniz cookies by the Hanover company Bahlsen! (The class voted for Leibniz, but not overwhelmingly.)
 - i. Struik called Leibniz a universal scholar—he did pioneering work in
calculus combinatorics logic linguistics philosophy etc.
and
“sought a universal method by which he could obtain knowledge, make inventions, and understand the essential unity of the universe”.

That quest led him to claim—more or less—

“that there is no effect without a cause; ... things cannot be otherwise than as they are; for as all things have been created for some end, they must necessarily be created for the best end.”

Leibniz became an early icon of French–German rivalry. Voltaire lampooned him marvelously, contrasting Leibniz’s “best of all possible worlds” philosophy with all the obvious misery on Earth. Leibniz was the model for Dr. Pangloss in Voltaire’s 1759 novella *Candide*. I confess: the previous quote is from its first chapter! Leonard Bernstein and Lilian Hellman connected the lampoon back to philosophical inquiry: the first and last lines of their 1956 Broadway musical comedy *Candide* are *Any Questions?*

- j. Watch: this French and German rivalry keeps recurring in mathematics!
4. *Struik*, sections 7.1–7.5
- a. Leibniz, through his work as adviser, diplomat, scholar, and tutor to the German elite, traveled to London and Paris and had numerous connections with their scientific communities. He was elected to the Royal Society in London early in life and to the Paris Academy late in life. He spent the last half of his life in Hannover, away from those communities, but he exerted major influence through his travels and his writings in their journals and his own. He was a major force behind the establishment of the academies of science in Berlin, St. Petersburg, and Vienna (Prussia, Russia, and Austria).
- b. At that time mathematics was centered in these academies, not in universities. The academies were instruments of nationalism and imperialism as well as temples of knowledge.



Leibniz's most influential followers were in Paris and in Basel. (Correction to Struik: by that time, German-speaking Basel was part of the Swiss Confederation, not a free Empire city.)

- d. The Bernoulli brothers in Basel developed much of the now familiar calculus, both theoretical and applied: infinite series, ODEs, PDEs, calculus of variations.
- e. [Click here](#) for some pictures of Basel. (It looks gloomy, but is not!)
- f. Their student Euler and three younger Bernoullis found employment in Russia, where the Europhile (even Germanophile) Peter the Great had set up the St. Petersburg Academy along lines suggested earlier by Leibniz.
- g. Euler traveled overland from Basel to Lübeck in 1727, and proceeded by ship to St. Petersburg. [Click here](#) to see what Euler saw in Lübeck. It was a much richer city than Basel.
- h. [Click here](#) to see what Euler saw in St. Petersburg, and some of his handwritten notes preserved in the archives there. Some are in Latin, apparently in preparation for publication of a calculus text. Some are more private, apparently notes in German on elementary geometry. He tended to write for himself in a mixture of these, using Latin for technical terms. The archives also have some of his notes written in French. I don't think these materials have been published in any way yet. The Russians are surprisingly lax at letting people like me touch them!
- i. During a period of Germanophobic unrest in Russia, Euler moved to the Berlin Academy, which had recently been reorganized by Frederick the Great. It had been founded by his father following Leibniz' advice. Euler did a prodigious amount of work managing it, but was personally incompatible with Frederick (a Francophile whose native language was Polish) and was never appointed director. During that time Euler continued very close ties with St. Petersburg. Finally, when the German Catherine the Great became leader in Russia, he returned, to remain for the rest of his life.