

1. *Struik, sections 5.8–5.9*
 - a. Struik noted that the French mathematician François Viète cleaned up and elaborated the Italians' solutions of the cubic and quartic equations, and made major advances in algebraic notation to do so, including introducing the use of variables. Struik spoke of the *irreducible case* of the cubic, which I should explain. (Again, reasoning at that time wasn't abstract enough to express the argument this way, but those people knew the basic facts.)
 - b. A cubic equation with real coefficients always has at least one real root. Moreover, if it has a non-real complex root z , then the conjugate of z is also a root. So the equation has three distinct real roots, one simple real root and one double real root, one triple real root, or one simple real root and two distinct conjugate complex roots. The solution procedure that I outlined ([click here](#)) produces a list of six possible complex roots, at most three of which are distinct. If the quantity under the square-root sign is negative, those will be real roots, but you have to do algebra with complex numbers to see that. Only in 1890 did Vincenzo Mollame, later a colleague of Mario Pieri, show that use of complex algebra is actually *required* for this: there is no short cut.
 - c. Struik also emphasized the development of logarithms in Scotland around 1600, by John Napier and Henry Briggs. (The former was an ancestor of the Napier for whom the New Zealand city was named.) I think Struik missed their simultaneous development in what is now the Czech Republic. These led to great improvements in many applications. I believe the first was in banking: constructing tables for amortization, present values, etc., requires difficult computations with powers and roots. But the use of logarithms in trigonometry became so vital that even down to my school years, we had to become highly adept at the use of logarithm tables *before* we entered the trigonometry course. Trigonometry texts usually included logarithm tables as an appendix.
2. *Struik, sections 6.1–6.2*
 - a. *Rechenhaftigkeit* = susceptibility to solution by computation.
 - b. I once read a wonderful book by Derek de Solla Price on the evolution of technology. He stressed the advances in the production of fine tools, especially lathes. It required a good lathe to make the parts of a better one, which was required to make the parts of a yet better one, etc., and with each step gear manufacture, hence clockwork manufacture, improved. [Click here](#) for photos of the magnificent 16th-century astronomical clock in St. Mary's Church in Lübeck, Germany. Then reflect on the enormous wealth displayed by that church and its surroundings, and on the significance of locating such a clock within the church across from city hall.
 - c. *Parabolic orbit*: you learn in calculus that the path of a projectile hurled obliquely upward is parabolic. Galileo discovered that experimentally.

- d. *Catenary* = curved shape assumed by a uniformly laden chain (*catena* in Latin). I believe it's dependent on exactly what you mean by uniformly laden, but with no load I think the curve is the graph of the hyperbolic cosine, which hadn't been thought of at that time.
- e. I mentioned earlier that the first mathematics literature in a "popular" European language was in Catalan. Most learned works in Europe were written in Latin, which all scholars knew. (We'll see that Peano wrote sometimes in Latin as late as 1889!) Struik stressed the emergence of literature in English, Dutch, and French, and mentioned some works related to geometry and art that were written in Italian and German. There were major differences in spoken dialects in these languages, down to modern times. In each case, the written language only approximated one of the dialects.
- f. Struik noted that around 1600 several mathematicians *anticipated results that we classify as calculus*, sometimes using faulty reasoning to get correct results. This sort of mathematics appeared first with the Greeks; when successful, the new methods were easier to describe and apply than their earlier ones. I forgot to include a particularly nice example when we discussed Archimedes. His derivation of the formula for the area under a parabola went something like this.

The area we now call $\int_0^a x^2 dx$ is approximately the sum S_n of the areas of rectangles with bases determined by the points $x_i = ai/n$ for $i = 0, \dots, n$ for some (large) integer n , and whose altitudes are the ordinates of the corresponding points on the parabola:

$$S_n = \sum_{i=1}^n \frac{a}{n} x_i^2 = \frac{a}{n} \sum_{i=1}^n \frac{a^2 i^2}{n^2} = \frac{a^3}{n^3} \sum_{i=1}^n i^2.$$

I don't know how Archimedes evaluated the sum, but there are many ways that involve only exceedingly elementary geometry (search for "proof without words sum of squares" or the like in *JSTOR*). Here's a way that uses an algebraic trick. You have to know that

$$\begin{aligned} 1 + 2 + \dots + n \\ &= \frac{1}{2}[(1 + n) + (2 + (n - 1)) + \dots + ((n - 1) + 2) + (n + 1)] \\ &= \frac{1}{2}n(n + 1) = \frac{1}{2}n^2 + \frac{1}{2}n. \text{ Next,} \end{aligned}$$

$$(i + 1)^3 = i^3 + 3i^2 + 3i + 1$$

$$i^2 = [(i + 1)^3 - i^3 - i - 1]/3$$

$$\begin{aligned} S_n &= \frac{a^3}{3n^3} \left(\sum_{i=1}^n (i + 1)^3 - \sum_{i=1}^n i^3 - \sum_{i=1}^n i - \sum_{i=1}^n 1 \right) \\ &= \frac{a^3}{3n^3} \left(\sum_{i=2}^{n+1} i^3 - \sum_{i=1}^n i^3 - \frac{1}{2}n^2 - \frac{1}{2}n - n \right) \end{aligned}$$

$$\begin{aligned} &= \frac{a^3}{3n^3} \left((n+1)^3 - 1 - \frac{1}{2}n^2 - \frac{3}{2}n \right) \\ &= \left(n^3 + \text{a quadratic polynomial in } n \right). \end{aligned}$$

The approximation gets ever closer as n increases, and this expression gets ever closer to $\frac{1}{3}a^3$, so that must be the formula for the area.

A short algebraic study reveals that this same technique will yield the areas under the graphs of higher integral power functions. As algebra improved, more integral calculus was invented.

3. *Quiz 2* was returned with a hardcopy debriefing.