

1. *Paper 1 guidelines*, continuing the discussion in lecture 3.
 - a. There are good style guides, available at bookstores.
 - i. I use Chicago 1993, the *Chicago Manual of Style*.
 - ii. You probably want something that says it's consistent with that but is designed for students, not professional writers.
 - b. Content
 - i. State your topic or problem in one or more introductory paragraphs. Then include the full discussion. That should determine the length of the paper. I do not impose arbitrary limits. There should be one or more paragraphs that clearly constitute a closing; their organization will probably differ depending on the topic.
 - ii. It is often helpful to think of telling a story, as I noted in lecture 3.
 - iii. Documenting sources is a part of the *content* of a paper. *I must be able to backtrack what you did.*
 - c. Presentation
 - i. The paper should be pleasant to read. I should not have to fight it to find out what you did.
 - ii. Force yourself to write an introduction that tells what you're about to do. Rewrite it later when you think the paper is completely drafted.
 - iii. Read your paper out loud. Read it to a friend. That way you should catch many blunders and diction errors. At the end re-read the introduction and ask yourself whether you covered what you said you would.
 - iv. I expect to see a bibliography. When you use a source in your paper, cite it in a remark in the text or in a footnote or endnote, which in turn refers to your bibliography. The course bibliography is in the format that I'm using now for writing books. It's a variant of the *Chicago* style. A more elaborate example is the bibliography of Marchisotto and Smith 2007. That contains more information than you need to supply; but it has an example entry for almost any type of source you might use.
 - v. The bibliography should list all and only those sources cited in the body of the paper. Don't pad it by listing things you merely read for background. If I need to know about something to backtrack you, you must cite it in the body text, which will trigger a bibliographic entry.
 - vi. The apportionment of information between body-text citations and bibliography is demonstrated nicely in Marchisotto and Smith 2007, section 1.1. *Almost no information is repeated.* (That way, when you change some detail, you only change it once; and that eliminates many chances for blunders.)
 - vii. I expect a typed paper. If you think you'll have trouble with symbols, ask me. Your word processor can probably do most of them. Through the Department you may have free access to *MathType*, which I use for some

formulas you see on the course website. If you can't deal with that, try marking formulas in by hand. (And please proofread: I can't read empty space.)

- viii. I don't care whether the paper is double-spaced. Don't pad length by double-spacing. If double-spacing makes it easier to read, fine.
 - ix. You can include illustrations, but I warn you, don't be tempted to spend great amounts of time making them superneat. If you need to add notation, doing it by hand is ok. Use your time on the history and organizing the paper.
 - x. Learn to use a spell checker even in the context of many words of non-standard English. Then *do* use it.
 - d. I expect to read these papers and comment on them *in extreme detail*.
 - e. Do not expect to get these back: you'll get a page or more of comments that you can compare with the copy (probably the original) that you will have kept for your records.
2. *Seminar on actuarial work*. Here, 11 March. [Click here](#) for a flyer.
 3. *Undergraduate Mathematics Conference*
 - a. An Undergraduate Mathematics Conference will be held at Sonoma State University on Saturday, April 4. Undergraduate students will give and/or attend fifteen-minute presentations about their projects. These can include undergraduate research or any other independent work that goes beyond the standard course curriculum. The conference will conclude with a talk, "Tropical Mathematics," by Berkeley professor Bernd Sturmfels, tailored toward undergraduates. (He was our Prof. Hosten's PhD research supervisor, and we have sent several alumni to him for PhD work. He is a good speaker!)
 - b. [Click here](#) for a two-page flier about the conference.
 4. *Area, concluded*. Ms Fick derived the formula for the area of a triangle from that for the area of a rectangle.
 5. *Volume, continued*
 - a. After elementary results about prisms, Euclid aimed for an equivalent of our formula for the volume of a triangular pyramid $\mathcal{P}$. That required some interesting preliminary steps.
 - b. First, the midpoints of the edges of each of the faces of $\mathcal{P}$ determine four planes, which dissect it into an antiprism, in the center, and four congruent subpyramids similar to $\mathcal{P}$, in the corners. Select any two of the subpyramids. Together they form less than half of $\mathcal{P}$. The remaining region can be dissected into two prisms. [Click here](#) to see a drawing.
 - c. Next Euclid proved proposition XII.5: two triangular pyramids are equal [in volume] if they have congruent altitudes and their bases are equal [in area]. Here is his proof, which uses the *method of exhaustion*.
 - i. Suppose [the volume of] the second, $\mathcal{V}_2$, exceeds [that of] the first, $\mathcal{V}_1$.

- ii. Dissect $\mathcal{V}_2$ into pyramids and prisms as in (1.b), and repeat the dissection recursively on all the pyramids obtained, until [the volume of] the aggregate $\mathcal{P}_2$ of all the pyramids is smaller than $\mathcal{V}_2 - \mathcal{V}_1$.
- iii. Euclid had prepared the way for that step by noting in X.1 that successively removing more than half of any quantity will eventually result in something smaller than any given quantity. This principle, or a closely related one, is often called *Archimedes' principle*.
- iv. Now dissect $\mathcal{V}_1$ recursively in exactly the same way, and denote by $\mathcal{P}_1$ [the volume of] the aggregate of all the pyramids that result. Euclid noted that the prisms in the first dissection correspond bijectively with those in the second, and by (1.b) [the volume of] each of the prisms in the first dissection is equal to [the volume of] the corresponding one in the second. Thus [the volume of] the aggregates $\mathcal{Q}_1$ and $\mathcal{Q}_2$ of all the prisms in the two dissections are equal.
- v. $\mathcal{V}_1 = \mathcal{P}_1 + \mathcal{Q}_1 = \mathcal{P}_1 + \mathcal{Q}_2 = \mathcal{P}_1 + \mathcal{V}_2 - \mathcal{P}_2$, hence
- vi. $\mathcal{P}_2 = \mathcal{P}_1 + \mathcal{V}_2 - \mathcal{V}_1 > \mathcal{V}_2 - \mathcal{V}_1$: contradiction!
- vii. Thus $\mathcal{V}_2 \neq \mathcal{V}_1$. Similarly, $\mathcal{V}_1 \neq \mathcal{V}_2$, hence $\mathcal{V}_2 = \mathcal{V}_1$, q. e. d.
- d. Let $\mathcal{P}$ be a triangular pyramid with apex A , and $\mathcal{Q}$ be the triangular prism with the same base, whose vertices include A . You can dissect $\mathcal{Q}$ into $\mathcal{P}$ and two triangular pyramids that by (1.c) are equal [in volume] to $\mathcal{P}$. [Click here](#) to see a drawing.
- e. *Thus [the volume of] a triangular pyramid is equal to one-third of [the volume of] a triangular prism with the same base.*
- f. This result can be extended to apply to a polygonal pyramid by dissecting its base into triangles and considering the corresponding triangular pyramids.
- 6. *Platonic solids.* The culmination of Euclid's geometry was in book XIII: the marvelous theory of the five Platonic solids. I discussed their construction in Smith 2000, section 8.4, and used that mathematics to draw the illustrations for that book. [Click here](#) for the screen dumps from which those were made. (The colors were chosen with respect to the way a monochrome printer rendered them after taking the complements of the color codes.)