

1. *Quiz 1 next Tuesday, 10 February*
 - a. Its purpose is to check that you are acquiring many of the main points covered by the text and lectures. I will *not* ask you to synthesize, putting these items together to give a coherent larger picture. That is for your papers, and for later study. Rather, the quiz will consist of several questions that should require only a couple of minutes each to answer.
 - b. I'll construct it by going through my notes and the Kennedy and Struik texts, stopping at whatever point we have reached by the end of this lecture. I'll jot down individual items worthy of questions. Then I'll construct some questions of the following types, to require twenty-five minutes:
 - i. Identify a given concept in one sentence or less.
 - ii. Explain a given idea in two sentences or so.
 - iii. Arrange some events or trends in chronological order.
 - iv. Match some people or groups of people with places or eras, or with concepts or techniques they're associated with.
 - v. Select among alternative short explanations of some concept.
 - vi. Select among alternatives for the context and purpose of developing a given part of mathematics.
 - c. For this quiz, that list may seem too elaborate, because there aren't that many feasible questions so early in the course. This same description will be used for the following quizzes, too.
 - d. Struik occasionally made controversial statements. If you wish to disagree with him, you need to say that you are doing so and give a reason.
 - e. The best way to prepare for a quiz is to make one yourself, following my plan.
2. *Struik, chapter 3*. He noted these trends that were occurring by the dawn of the Iron Age, around 900 B.C.E.
 - a. Replacement of bronze by iron was cheapening production and increasing trade and participation of the population in public affairs. Those were further stimulated by the introduction of alphabetic writing and of coined money.
 - b. Traditional science stagnated in Egypt and Mesopotamia, but new vigor appeared in the trading towns of the central and eastern Mediterranean. These independent but intercommunicating merchant communities made effective use of the new technologies and inventions.
 - c. They had no established religious-political hierarchy, and created enough wealth, that some individuals were free to consider philosophical and mathematical questions, to ask *why* their mathematics worked, and to organize their inquiries into nature as sciences.
 - d. Little historical record survives from this time, but there has been enough for painstaking scholars to analyze and partially describe the scope of the achievements of the era.

- e. Struik dwelt on the knowledge accumulated by the community in southern Italy founded by Pythagoras (circa 570–circa 490 B.C.E.), and by scholars in the region between Greece and Turkey from that time through about 400 B.C.E., leading up to the Golden Age of Greece.
 - f. Some of their mathematical results are connected with logical arguments, but for others the deductive threads are missing or confused.
3. Let's consider some of these.
- a. As mentioned earlier, the general formula for the volume of a pyramid was known by this time. You can see easily that the volume of a pyramid formed from a cube by joining the four vertices of one face to the midpoint of the cube is one sixth of the volume of the cube, hence one third of the base area of the pyramid times its height. I think similar arguments can be used for some other pyramids, but not in general. How the general formula was discovered is a puzzle, and it was certainly not proved until close to Euclid's time, around 300 B.C.E.
 - b. That an angle inscribed in a semicircle is right is often called *Thales' theorem*. Here's a proof in the style of Euclid, but more precise with regard to sides of a line, so that you can draw the figure from the description.
 - i. Consider a semicircle ACB with center O ; it circumscribes $\angle ACB$.
 - ii. $OA = OC$; thus the isosceles triangle theorem yields $\angle OCA \cong \angle OAC$.
 - iii. Consider line $a \parallel OC$ through A ; let D be any point of a on the same side of line AB as C .
 - iv. By the alternate-interior-angles theorem, $\angle DAC \cong \angle OCA$.
 - v. $OB = OC$; thus the isosceles triangle theorem yields $\angle OCB \cong \angle OBC$.
 - vi. Consider line $b \parallel OC$ through B ; let E be any point of b on the same side of line AB as C , and F be any point of b on the other side.
 - vii. By the alternate-interior-angles theorem, $\angle EBC \cong \angle OCB$.
 - viii.
$$\begin{aligned} 2m\angle ACB &= 2(m\angle OCA + m\angle OCB) \\ &= 2m\angle OCA + 2m\angle OCB \\ &= (m\angle OCA + m\angle OCA) + (m\angle OCB + m\angle OCB) \\ &= (m\angle OAC + m\angle DAC) + (m\angle OBC + m\angle EBC) \\ &= m\angle OAD + m\angle OBE. \end{aligned}$$
 - c. By transitivity of parallelism, $a \parallel b$.
 - i. By the alternate-interior-angles theorem, $\angle OAD \cong \angle OBF$.
 - ii. Therefore $2m\angle ACB = m\angle OBF + m\angle OBE = 180^\circ$: $\angle ACB$ is right.
 - d. Thales may have known this around 600 B.C.E., but the proof technique came only a couple of centuries later.
 - e. You can compute the *mean proportional* x between given lengths a and b ($a:x = x:b$) with ruler and compass:
 - i. locate collinear points A, B, C so that $AB = a$ and $BC = b$.
 - ii. The perpendicular to AC at B intersects the circle with diameter AC at a point D ; let $x = BD$.
 - iii. Then $\angle ADC$ is right, $\triangle ADB \sim \triangle DBC$, and hence $a:x = x:b$.

- f. Note that $x = \sqrt{ab}$, a quantity that figured in the formula for the volume of a frustrum of a cone. The mean proportional appears quite often in geometric problems. You can compute $x = \sqrt{a}$ with ruler and compass by setting $b = 1$ in the previous example.
- g. *Double proportionals*: given a, b find x, y with $a:x = x:y = y:b$. This would yield $y = \sqrt[3]{a}$ by setting $b = 1$. Can it be done with ruler and compass?
- h. The Greeks regarded the three *Delian* problems as major challenges: to construct with ruler and compass
- the side t of a cube with twice the volume of a cube with an arbitrarily given side s ,
 - a line trisecting an arbitrarily given angle, and
 - the side of a square with the same area as an arbitrarily given circle.
- i. The first could be solved with a double proportional, since $t = s\sqrt[3]{2}$. All three problems spurred much research until mathematicians discovered during the 1800s that none of those constructions can be carried out with ruler and compass alone.
- j. They create a great annoyance for academic mathematicians, since many talented amateurs become highly intrigued by the complicated geometry and resentful of the negative pronouncements of the authorities. The theory behind the negative results is more advanced, slightly beyond our Math 335 and 370. Ruler-and-compass zealots often don't understand it, and frequently submit to university mathematicians purported solutions of the Delian problems, which of course must contain some errors. Our need for good public relations suggests that we should not ignore such submissions, and unfortunately it can take days to find the error in a multipage document.
- k. Struik suggested that the Pythagoreans may have known some of the properties of the regular polyhedra. He mentioned the occurrence of almost-regular dodecahedra in pyrite crystals, and of regular dodecahedral objects in excavations of sites dating to those times. [Click here](#) for some examples.
4. *Axiomatic method*
- It should be clear that by about 400 B.C.E. any systematic presentation of the known mathematics would have required sophisticated organization.
 - The scene is set for the development of the axiomatic method, and its partial implementation in Euclid [1908] 1956, written around 300 B.C.E.
 - [Click here](#) for a discussion of this topic, section 2.3 of J. T. Smith 2000. I have copied its bibliographical references into this course's bibliography.
 - Relationships:

Socrates	Thales	
Plato	Pythagoras	
Aristotle	Archytas	
Alexander the Great		others
Ptolemy and the founding of Alexandria		
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	Euclid	

- e. I don't think Struik considered the history of the axiomatic method adequately. But I won't write about it here: I'd merely be repeating J.T. Smith 2000.
- f. The method, as expounded by Aristotle, underlies virtually all presentations of modern mathematics, particularly those you study in algebra courses here.
- g. In the past, Aristotle's guidelines weren't followed as closely as now—certainly not by Euclid. Ascertaining just what Euclid and his earlier transcribers intended is a puzzle still studied by historians.
- h. What is needed for modern mathematics—a method virtually identical with Aristotle's original prescription—reemerged only around 1890. Our hero Peano was instrumental in that development.