

1. The principal source for biographical articles on noted mathematicians is Gillispie 1970, the wonderful eighteen-volume *Dictionary of Scientific Biography*. Its articles were written by world-known experts. It's available in the Library Annex down near Lake Merced Boulevard. (Suggestion: that large study area has wireless Internet access and may be underutilized! The reference librarians also work there.)
2. The following books are on *two-day reserve* for this course. They are available at the Library Distribution Center, HSS 102. These will provide general information on history topics. Some are much more comprehensive than Struik.
 - a. Bell 1937: *Men of Mathematics*. A collection of entertaining but sometimes misleading biographies.
 - b. Boyer and Merzbach 1989: a comprehensive history of mathematics textbook.
 - c. Burton 2007: the current leading expensive comprehensive history of mathematics textbook.
 - d. Eves 1990: a comprehensive history of mathematics textbook.
 - e. Grattan-Guinness 1997: the most recent comprehensive history of mathematics. A reference book, not a textbook.
 - f. Struik [1948] 1987: the course text.
3. *Paper 1*
 - a. Consider the *deadlines* on the schedule of class [meetings](#).
 - b. I will *not* accept late papers, because I expect to drive after class to my home in Siskiyou County and grade papers there. I expect to read these and comment on them *in extreme detail*.
 - c. You should *consult with me about your paper topics* well before they're due. Some of you have started that already.
 - d. Your *written proposal* should contain the following items.
 - i. Title of your topic (not of your paper).
 - ii. Some idea of the scope of your topic.
 - iii. How you thought of it. (This can sometimes help me help you.)
 - iv. What source material(s) are you starting your search with?
 - v. What further source materials are you likely to find and use?
 - vi. What help might you need?
 - e. Later, I'll issue some guidelines on constructing a paper. Two are worth discussing now.
 - i. Documenting sources is part of the *content* of a paper (not just its presentation). *I must be able to backtrack what you did*. You must *keep track* of what you're looking at, else you won't be able to document it later.
 - ii. Think of *telling a story*.
 - (1) That provides a structure for you to hang a paper on.
 - (2) And it gives you a criterion for inclusion/exclusion of material:
 - (a) include necessary background material,
 - (b) and what's needed to tell the story,

- (c) and maybe some nice embellishments,
- (d) but exclude other stuff.

4. *Grading*

- a. *Term papers.* The last time I taught this course, I used the following scheme:

Appropriateness:	5	points
Scope:	5	
Originality:	3	
Presentation:	6	
<i>Total:</i>	19	points

This is tentative, but approximately right.

- b. *Course.* The course has 100 percentage points, distributed between papers, participation, and quizzes as indicated in the syllabus. In recent years, generally but not always, the following intervals have corresponded approximately to my A,B,C,D,F grades: 80–100, 65–79, 50–64, 40–49, 0–40. (Sometimes I must correct for unforeseen circumstances.)
5. *Peano.* We've followed Peano through his schooling; he's ready to enter university. We'll now turn to Struik to survey the history of what Peano, and we, studied in school. We'll come back to Peano later.
6. *Reviews of Struik.*
- a. I reported in lecture 1 on the review Boyer 1949.
 - b. The MSN review by Dijksterhuis gives little beyond a factual description, but does mention Struik's emphasis on the interplay between mathematics and society.
 - c. Thanks to Mr. Baravati for reporting on three more reviews.
 - i. Dehn noted in 1955, page 643, "This work has the great advantage of being written by an accomplished mathematician as well as a reliable historian." For Boyer, that was evidently irrelevant. Dehn was a rather famous research mathematician, Boyer was not.
 - ii. In 1989 Rowe wrote, "...the literature surveying the history of mathematics has traditionally been dominated by works that take a narrow, internalist approach." He deplores that and regarded Struik as a breakthrough. Reviewing our later edition in 2001, Rowe heaped on even more praise. He mentioned Struik's 1942 work on the sociology of mathematics, his Marxist approach, and the tendency of Marxists toward dogmatism, then wrote (page 592), "Indeed, [Struik] was an ongoing effort to present an overarching portrait of mathematical developments on a broad canvas of time and space. Its spirit was open-ended, without a trace of dogmatism, and its Marxist features were so attenuated that few could have guessed this guiding orientation unless they either knew the author personally or had read his other work."
 - d. I haven't read Struik 1942 yet, but will do so and list it in the bibliography as soon as possible.
7. *Assignment.* Read Struik, chapter 2.

8. *Struik, chapter 1*

- a. The mathematics of “primitive” cultures—*ethnomathematics*—is a thriving research area now, shared between mathematics and anthropology. I learned a lot at a local ethnomathematics conference some years ago. Particularly notable were talks on
 - i. The evolution of number-words used by isolated tribes in Papua New Guinea as western commerce is changing their way of life.
 - ii. The design of traditional Laotian looms to store information about complex symmetry patterns.
 - iii. The adaptation of tribes in Brazil to the use of GPS techniques to locate boundaries and solve land disputes. This was given by SFSU anthropology professor Marianna Ferreira.
- b. Although ethnomathematics is very interesting, be careful: *this course is about history not anthropology*.
- c. Struik emphasized the emergence of paleolithic cave painting as evidence of the gradual understanding of 2D depiction of 3D objects.
- d. He mentioned that as early as neolithic times, to build structures and tools and to plan agriculture and trade required basic engineering, mathematical, and abstract language skills. Ornamentation from these times often shows sophisticated geometric knowledge.
- e. I know something about the use of ornamental symmetry in traditional arts and crafts. I used some Native American examples to illustrate J. T. Smith 2000. I’ll circulate that in class. [Click here](#) for two examples.
- f. Struik mentioned that rituals often had mathematical components. He discussed astronomy and time measurement together, because the former is often used in the latter. These certainly had to do with religion, but also with navigation and trade.
- g. Struik listed some civilizations that developed some mathematics that did not survive to influence later peoples (at least to our knowledge). And he noted that it is possible for a major civilization to practice some mathematics without a system of writing.
- h. English terms
 - i. *autochthonous* = aboriginal
 - ii. *codex* = ancient manuscript (plural *codices*)
 - iii. *cubit* = about half a yard
 - iv. *dyadic* = having to do with 2
 - v. *ell* = about a yard
 - vi. *fathom* = two yards
 - vii. *golden section* = $\frac{1}{2}(1 + \sqrt{5})$
 - viii. *mensuration* = measurement
 - ix. *numerology* = pseudoscience of the occult significance of numbers
 - x. *pentalpha* = pentagram (looks like 5 A’s)
 - xi. *polyhedron* = closed figure with flat polygonal faces
(impossible to define both precisely and briefly)

- xii. *quinary* = having to do with 5 ..
- xiii. *solstice* = beginning of summer or winter ...
- xiv. *triangular number* = the number of dots in an array like this:
- xv. *vigesimal* = having to do with 20

We have a special word for 20 of anything: *score*.

- i. Struik mentioned that the growth of a science does not always occur
in the same order as its instruction. Often applications come long
before theory. For example, artists knew about and applied complicated
properties of ornamental symmetry hundreds of years before the theory was
developed in the late 1800s for crystallography, then specialized in the 1920s
for ornaments.
9. *Frustra*. Struik will mention in chapter 2 that ancient Egyptians knew how to
compute the volume of a frustrum of a pyramid. Do you? To avoid frustration next
time, let's derive that formula.
- a. Consider a plane region R with area B , and a point X at a distance h from
that plane. The set C of points on segments between X and points of R
is called a *cone*. A pyramid whose square base has side b is the special case
of a cone with square R and $B = b^2$. You can think of the cone as consisting
of plane slabs of "infinitesimal" thickness dx parallel to R at distance x
from X . Plane geometry says that each such slab is similar to R , with their
linear measurements in the ratio x/h . Thus their areas have ratio $(x/h)^2$, and
the slab's area is approximately $(x/h)^2 B$. Thus its volume is approximately
 $(x/h)^2 B dx$. The volume of the cone should thus be $\int_0^h (x/h)^2 B dx = \frac{1}{3} Bh$.
 - b. A *frustrum* of C is the part of C between the base plane and a plane E
parallel to the base at a distance $x < h$ from it. The part of E in the cone
is called its *upper base*; call its area A . The volume V of the frustrum is the
difference between the volume of C and that of the cone with the same
vertex, base A and altitude $h - x$:

$$V = \frac{Bh}{3} - \frac{A(h-x)}{3}.$$

Because $B/A = [h/(h-x)]^2$, we have

$$\begin{aligned} h-x &= h \sqrt{\frac{A}{B}}, & x &= h \frac{\sqrt{B} - \sqrt{A}}{\sqrt{B}} \\ V &= \frac{Bh}{3} - \frac{Ah}{3} \sqrt{\frac{A}{B}} = \frac{h}{3\sqrt{B}} \left((\sqrt{B})^3 - (\sqrt{A})^3 \right) \\ &= \frac{h}{3} \frac{\sqrt{B} - \sqrt{A}}{\sqrt{B}} \left((\sqrt{B})^2 + \sqrt{AB} + (\sqrt{A})^2 \right) \\ &= \frac{x}{3} (B + \sqrt{AB} + A). \end{aligned}$$

- c. Since pyramids are cones in this sense, frustra of pyramids are frustra of cones; therefore this argument establishes the formula on Struik, 23.
- d. Deriving the formula for the volume of a cone or pyramid was easy, but involved calculus. Justifying that calculus would require a chapter of a current text, which would involve much algebra. Deriving from this formula the one for the volume of a frustrum required no calculus, but the algebra was troublesome. (It took me many tries to find the arrangement above.)
- e. The ancient Egyptians had neither calculus nor algebra.
- f. Over the centuries, mathematicians repeatedly asked whether this result could be derived without any use of calculus and with only minimal algebra. We shall return to that question several times.